

Optimizing Areas and Perimeters

Learning Goals

- determine through investigation which shapes to give largest areas and smallest perimeters
- use constrains to find optimal dimensions

Activity 1

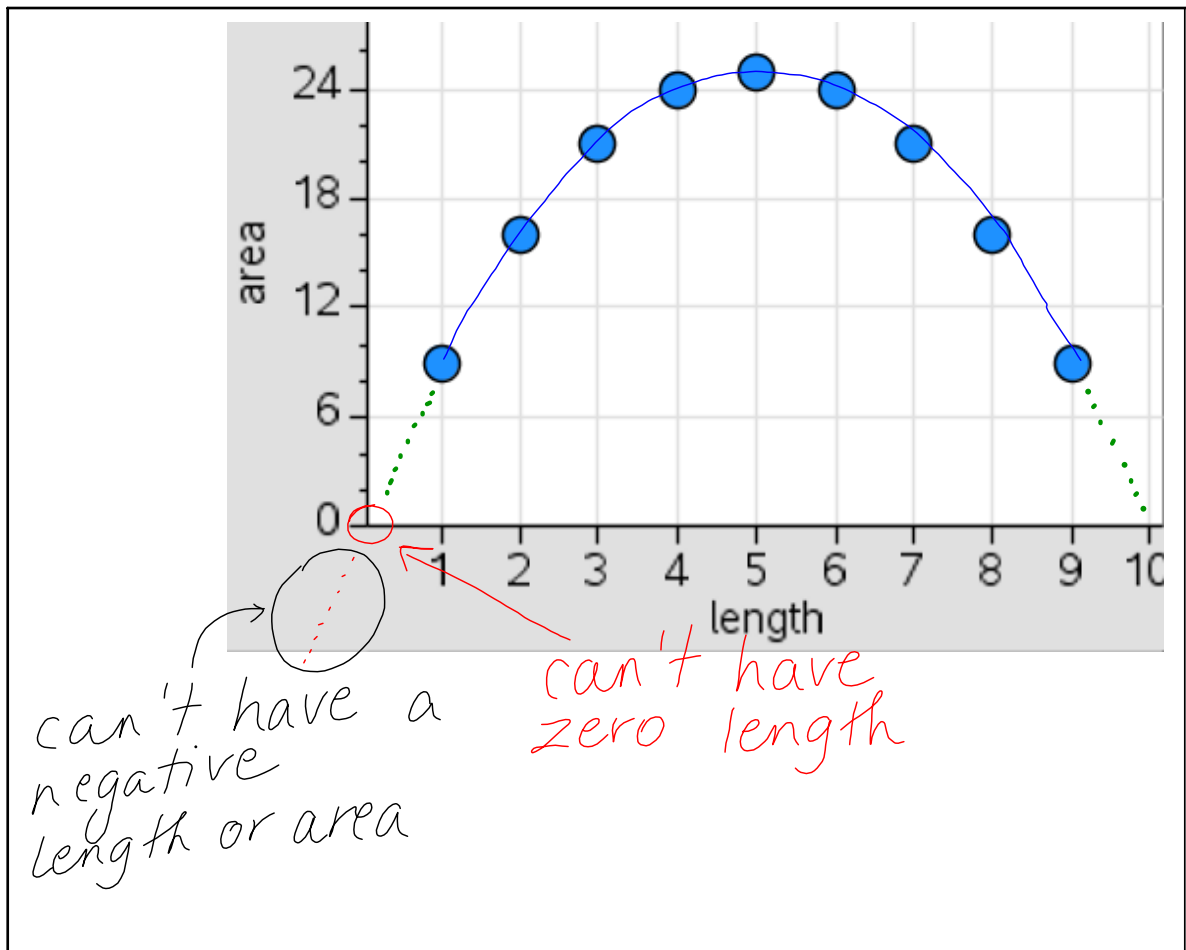
Fencing the Yard - Maximum Area for a Fixed Perimeter

FARMER BROWN'S PEN

Farmer Brown wants to build a rectangular pen for his chickens. He has **20 metres** of fencing available. Determine the maximum (largest) area, he can enclose by filling in the following chart. (use the data provided). Draw each of the pens on graph paper using the following scale: 1 side of a square represents 1 m.

1. Draw the diagrams.
2. Fill in the chart on the next page.
3. After you fill in the chart, create a graph of length vs. area.
4. After you made your graph, come back to this page and answer the questions.

Width	Length	Area	Perimeter
1	9	$lw = (1)(9) = 9$	$2l + 2w = 2(1) + 2(9) = 20$
2	8	16	20
3	7	21	20
4	6	24	20
5	5	25	20
6	4	24	20
7	3	21	20
8	2	16	20
9	1	9	20



Questions

1. What patterns do you see in the table?

width \uparrow perimeter —
 length \downarrow area $\nearrow \searrow$

2. Why did the first column stop at 9 m?

If the length is longer than we wouldn't have a rectangle.

3. What pattern do you see in the graph?



4. Which rectangular area is the largest? 25 m^2

What are the dimensions of this rectangle?

$$5 \text{ m} \times 5 \text{ m}$$



5. Which rectangular area is the smallest?

6. What are the dimensions of this rectangle?

$$1 \text{ m} \times 9 \text{ m}$$



7. Which one should Farmer Brown choose? 5×5

8. Why?

largest area

9. What is the optimal shape for the largest area?

square (or something very close)

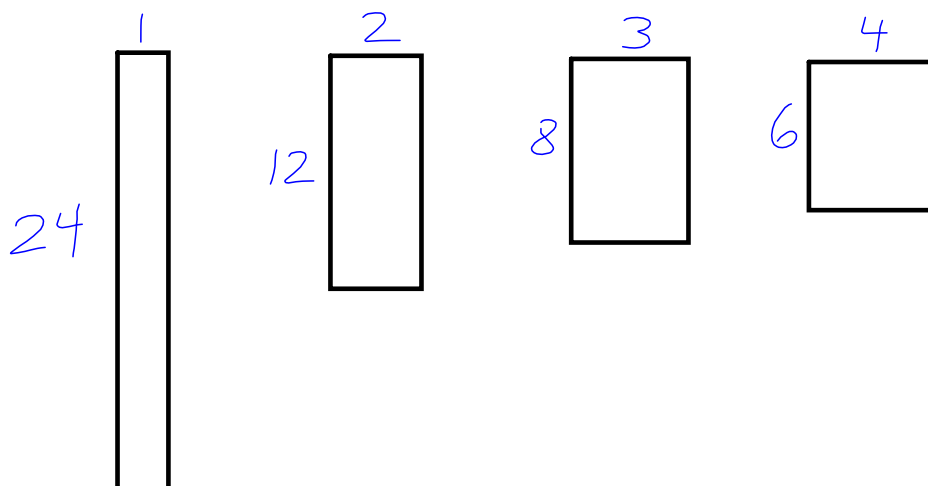
Activity 1: Farmer Brown's Pen**Conclusions**

When the **perimeter is given**, to get the largest area we need a shape that is a **square** (or close to it)

Activity 2:**Mrs Jones's Gardening Shed**

Mrs. Jones would like to build a garden shed with a rectangular shape. The shed must have an area of 24 m^2 . Mrs. Jones wants to keep the cost as low as possible. Each meter of fencing costs \$25.00.

Draw 4 different rectangles with areas of 24 m^2 .



Use your rectangles to fill out the chart. Show all your work using formulas.

Length	Width	Perimeter	Area	Cost of Fencing
24	1	$2(24) + 2(1)$ $= 50$	$(1)(24)$ $= 24$	$50(25)$ $= 1250$
12	2	28	24	700
8	3	22	24	550
6	4	20	24	500

1. What is special about the numbers in the width column?

Divide in 24 evenly

2. What are the dimensions of the rectangle with the largest perimeter?

$$24 \times 1$$

3. What is the perimeter of that rectangle?

$$50 \text{ m}$$

4. What are the dimensions of the rectangle with the smallest perimeter?

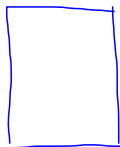
$$6 \times 4$$

5. What is the perimeter of that rectangle?

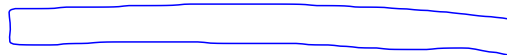
$$20 \text{ m}$$

6. What do you notice about the shape of the rectangles with the largest and smallest perimeters?

smallest



Largest



7. Which shape should Mrs. Jones choose? Justify your answer.

6×4 it has the shortest perimeter

Activity 2: Mrs. Jones' Gardening Shed

Conclusions

When **area is given**, to find the shortest perimeter we need a shape that is short and fat.

So we are looking for a **square** or a rectangle very close to it.

Jane wants to have a rectangular garden with an area of 42 m^2 . She wants to put a fence around it. What dimensions should the garden be to use the least amount of fencing?

least amount of fencing?
 ↖ minimum perimeter

1. We want a square.

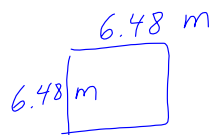
2. $A = lw$ ← area is given
 ↘ same

$$A = l(l)$$

$$\sqrt{42} = \sqrt{l^2}$$

$$6.48 = l$$

∴ square



Example:

There are 30 pieces of 2 m fencing. A farmer would like to fence the largest possible area for his chickens. What are the dimensions of the largest area?

- Fence can not be cut
- Fence can be cut
- He wants to put it beside a ~~river~~ house

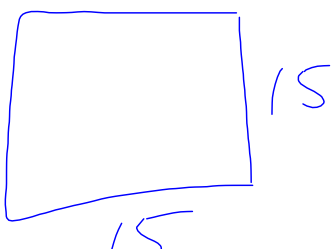
a.

Total fence available

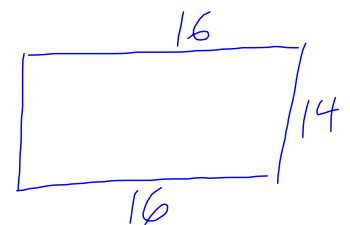
$$30(2) = 60$$

$\frac{60}{4} = 15$

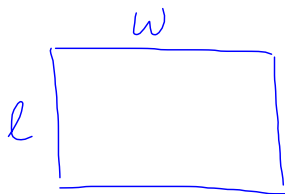
~~square~~



~~have to cut the fence~~



b.



$$P = 2l + 2w$$

$$60 = 2l + 2l \leftarrow \text{square}$$

$$60 = 4l$$

$$\frac{60}{4} = \frac{4l}{4}$$

$$15 = l$$

$$\therefore 15 \text{ m}$$

c.

If one side is not needed
than we are **not** looking
for a **square** shape.

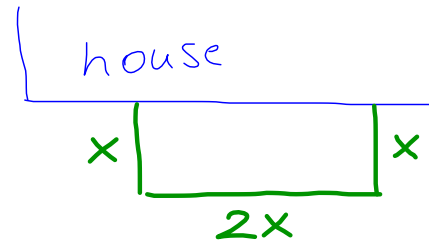
only 1
length
↓

$$P = l + 2w$$

$$60 = 2x + 2x$$

$$60 = 4x$$

$$15 = x$$



$$\therefore 15 \times 30 \text{ m}$$



On the Boards...

1. For each perimeter, what are the dimensions of the rectangle with the maximum area? What is the area?

a) 40 cm

b) 110 feet

c) 25 m

d) 87 inches

$$P = 4w$$

$$40 = 4w$$

$$10 = w$$

$$A = 10^2$$

$$A = 100 \text{ m}^2$$

$$P = 4w$$

$$110 = 4w$$

$$27.5 = w$$

$$A = 27.5^2$$

$$A = 756.25 \text{ ft}^2$$

2. For each area, what are the dimensions of the rectangle with the minimum perimeter? What is the perimeter?

a) 25 square feet

b) 81 m²c) 144 cm²

d) 169 square inches

$$A = lw$$

$$A = w^2$$

$$25 = w^2$$

$$5 = w$$

$$P = 4w$$

$$= 4(5)$$

$$= 20 \text{ ft}$$

$$A = lw$$

$$A = w^2$$

$$81 = w^2$$

$$9 = w$$

$$P = 4w$$

$$= 4(9)$$

$$= 36 \text{ m}$$

3. At an outdoor festival, 2-m sections of fencing are used to enclose an area for food sales. There are 100 sections of fencing available.
- How many metres of fencing are available altogether?
 - Determine the maximum rectangular area that could be enclosed.
How does the fact that the fencing is in sections affect your answer?

$$a.) \quad 2(100) = 200 \text{ m}$$

$$b.) \quad P = 4w$$

$$200 = 4w$$

$$50 = w$$

$$\begin{aligned} A &= w^2 \\ &= 50^2 \\ &= 2500 \text{ m}^2 \end{aligned}$$

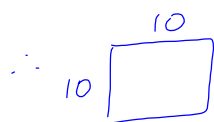
it is good because it is divisible by 2

4. A rectangular patio is to be constructed from 100 congruent square tiles.
- What arrangement of tiles would give the minimum perimeter?
 - Suppose each tile has side length 50 cm. What would be the minimum perimeter? What would be the area of the patio?

$$A = w^2$$

$$100 = w^2$$

$$10 = w$$



$$P = 2l + 2w$$

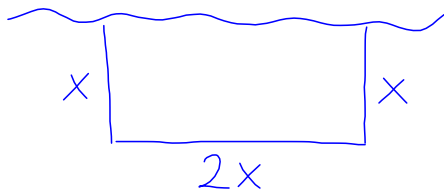
$$P = 4w$$

$$P = 4(50 \times 10)$$

$$P = 2000 \text{ cm}$$

$$\begin{aligned} A &= w^2 \\ &= (50 \times 10)^2 \\ &= 250000 \text{ cm}^2 \\ &= 25 \text{ m}^2 \end{aligned}$$

5. A lifeguard is roping off a rectangular swimming area using the beach as one side. She has 200 m of rope.
- a) Determine the greatest area she can rope off and its dimensions.



$$P = x + 2x + x$$
$$200 = 4x$$
$$50 = x$$

$$A = lw$$
$$= 50(100)$$
$$= 5000 \text{ m}^2$$

Seatwork / Homework

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