Unit 5: Graphincal Models

Trends in Graphs and Rates of Change

(5.1) Trends in Graphs

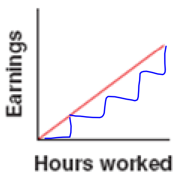
Using Graphs to Visualize Relationships

•A graph is a visual representation of the relationship between two quantities.

•It shows how one quantity changes with respect to the other.

Describing Relationships in Graphs

Jack's Babysitting Earnings



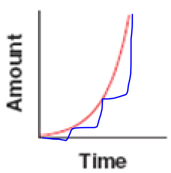
rise

Pairs of points with equal vertical distance have equal horizontal distance.

run

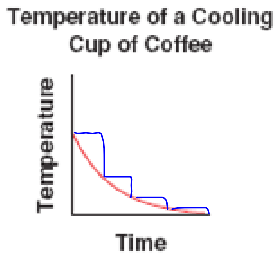
As the number of hours Jack works increases, his earnings increase by a constant amount.

Amount of a Compound Interest Investment



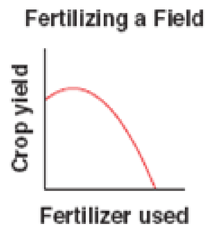
•The vertical distance between pairs of points with equal horizontal distances are increasing.

•The amount of the compound interest investment increases over time, slowly at first and then more quickly.



•The vertical distances between pairs of points with equal horizontal distances are decreasing.

•The coffee temperature decreases over time, rapidly at first, then more slowly, and finally leveling off at room temperature.

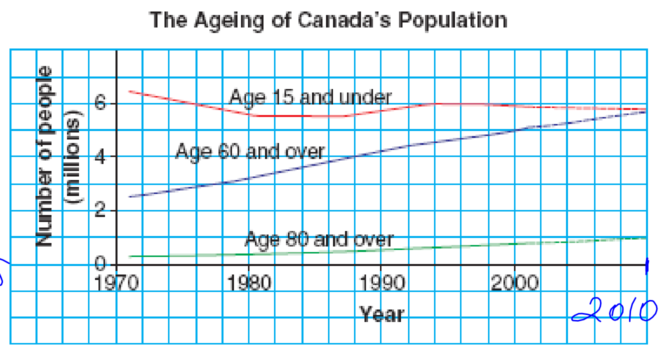


- The vertical distances between pairs of points with equal horizontal distances are increase, then decrease
- As fertilizer use increases, the crop yield increases, reaches a maximum, and then decreases

Trends in Graphs

Trends or patterns of change, in a graph are often used to justify decisions and make predictions.

Example:



a) Use the graph to predict the number of Canadians in each age group in 2010

- Under 15: 5.9
- 60 and over: 5.8
- 80 and over: 1

millions

b) What decisions might the Canadian government make in response to the trends in the graph?

- retirement homes ↑
- community events geared to older people ↑
- health care ↑

pension ↑

regional data for young people is more useful

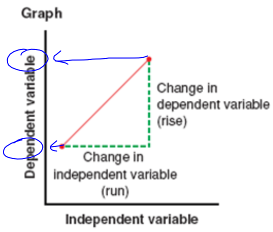
Rate of Change - the change of one variable relative to the change in another variable

ex. velocity = $\dfrac{\text{change of distance}}{\text{change of time}}$

Slope
- rate of change
- steepness of the line
- represented by m

Average rate of change

Average rate of change $= \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{\text{Rise}}{\text{Run}}$



Table

Independent variable	Dependent variable
x_1	y_1
x_2	y_2

NOTE: do not count the squares ⟶ use the scale

ex: Calculte the average rate of change.

Time (min) x	Height (m) y
0	2000
4	1400

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

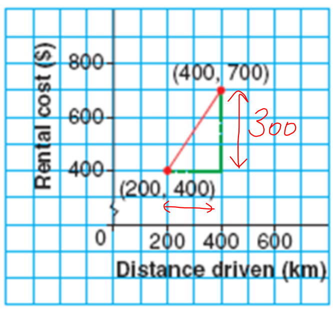
$= \dfrac{2000 - 1400}{0 - 4}$

$= \dfrac{600}{-4}$

$= -150$

Height decreases by 150 m/min.

Vehicle Rental Cost



$m = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{300}{200}$

$= 1.5$

$\therefore$ \$1.5/km

Comparing Rates of Change

- The distance required to stop a car depends on the speed at which the car is traveling.
- These tables show the reaction distance and braking distance needed to stop a car on dry pavement given speeds.

Speed (km/h)	0	10	20	30	40	50
Reaction distance (m)	0	2	4	6	8	10

Speed (km/h)	0	10	20	30	40	50
Braking distance (m)	0.0	0.5	2.0	4.5	8.0	12.5

Speed (km/h)	Reaction distance (m)	Change in distance / Change in speed
0	0	
10	2	$\frac{2}{10} = 0.2$
20	4	0.2
30	6	0.2
40	8	0.2
50	10	0.2

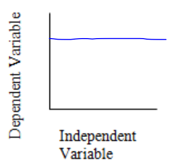
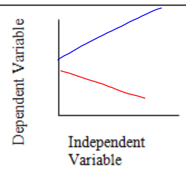
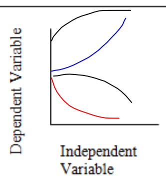
Speed (km/h)	Stopping distance (m)	Change in distance / Change in speed
0	0.0	
10	0.5	$\frac{0.5}{10} = 0.05$
20	2.0	$\frac{1.5}{10} = 0.15$
30	4.5	0.25
40	8.0	0.35
50	12.5	0.45

The rates of change are constant.
So, the reaction distance increases by 0.2 m for every 1 km/h increase in speed.

The rates of change are different.

Identifying Rates of Change

- When given a table, look for the rate of change by finding the slope.
- When given a graph, look for the rate of change by seeing the shape of the graph.

Rate of Change	Table	Graph
Zero	y-values are the same 1st diff are zero	
Constant	1st diff are the same	
Changing	1st diff are different	

On the Boards...

1. For each table, name the variables.

a)

Hours worked	Earnings ($)
4	32
20	160

b)

Pages printed	Cost ($)
1000	56
5000	145

c)

Distance driven (km)	Fuel used (L)
45	3
60	12

2. State the units of the rate of change for each table in question 1. What does the rate of change represent?

3. Refer to the tables in question 1. Determine the average rate of change between each pair of points in the table.

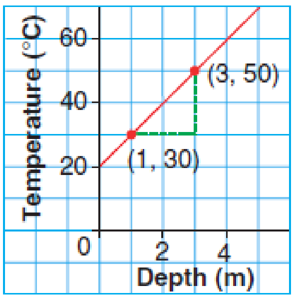
$m = \frac{160-32}{20-4}$ $\quad \frac{145-56}{5000-1000}$ $\quad \frac{12-3}{60-45}$

$= \frac{128}{16}$ $\quad = \frac{89}{4000}$ $\quad = \frac{9}{15}$

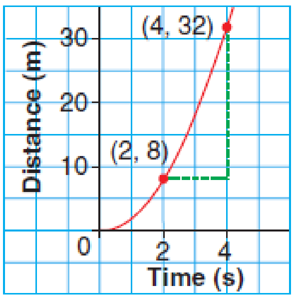
$= \$8/hr$ $\quad = \$0.022/sheet$ $\quad = 0.6\ L/km$

4. For each graph, name the variables.

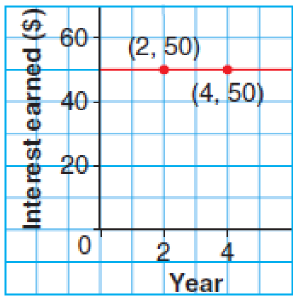
a) Graph A



b) Graph B



c) Graph C



5. State the units of the rate of change in each graph in question 4. What does each rate of change represent?

6. Refer to the graphs in question 4. Determine the average rate of change between the indicated points on the graph.

$\frac{20}{2}$ $\quad \frac{24}{2}$ $\quad \frac{0}{2}$

$= 10\ C°/m$ $\quad = 12\ m/s$ $\quad = \$0/year$

7. To save energy, an office building is only heated during business hours.

a) When is the temperature:

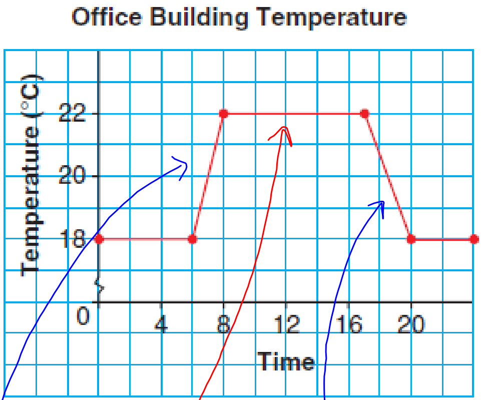
i) Constant? 8am − 5pm

ii) Decreasing? 5pm − 8pm

iii) Increasing? 6am − 8am

b) Calculate the rate of change during each time period from part a.

c) Describe the connection between your answers in parts a and b.



$\frac{4}{2} = 2\ {}^{\circ}C/h$ $\quad \frac{0}{9} = 0\ {}^{\circ}C/h$ $\quad \frac{-4}{3} = -1.33\ {}^{\circ}C/h$

Seatwork

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