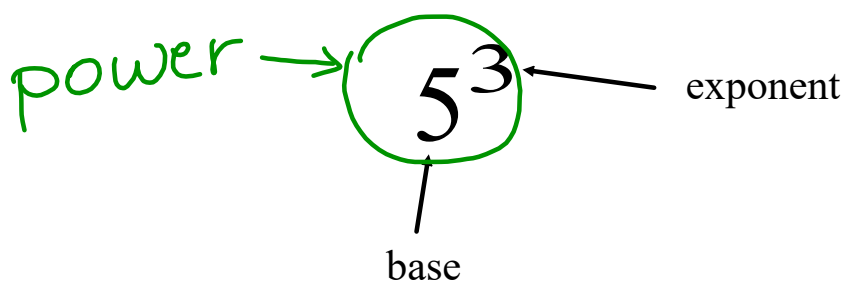


# Laws of Exponents

## Learning Goal

- review laws of exponents
- use laws of exponents to simplify algebraic expressions

What is a power?



Expand:

$$5^3 = 5 \times 5 \times 5$$

$$\frac{2}{3}^4 = \frac{2 \times 2 \times 2 \times 2}{3} \quad \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$$
$$= \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$$

### Multiplying Powers

Complete the following table:

Product of Powers	Product Form	Power Form
$3^5 \times 3^2$	$(3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3)$	$3^7$
$5^2 \times 5^4$		$5^6$
$x^3 \times x^2$		$x^5$
$a^2 \times a^4$		$a^6$
$(x^3y)(x^2y^2)$		$x^5y^3$

#### Summary:

To multiply powers with the same **base**, keep the base the same and add the exponents.

### Dividing Powers

Complete the following table:

Quotient of Powers	Product Form	Power Form
$3^5 \div 3^2$	$\frac{\cancel{3 \times 3 \times 3 \times 3 \times 3}}{\cancel{3 \times 3}}$	$3^3$
$5^5 \div 5^3$	$\frac{\cancel{5 \times 5 \times 5 \times 5 \times 5}}{\cancel{5 \times 5 \times 5}}$	$5^2$
$x^3 \div x^2$		$x$
$\frac{a^6}{a^4}$		$a^2$
$\frac{x^3y^2}{x^2y}$		$xy$

#### Summary:

To divide powers with the same **base**, keep the base the same and subtract the exponents.

### Powers of Powers

Complete the following table:

Power of Powers	Product Form	Power Form
$(5^2)^3$	$5^2 \times 5^2 \times 5^2$	$5^6$
$(2^4)^2$	$2^4 \cdot 2^4$	$2^8$
$(x^3)^4$	$x^3 \cdot x^3 \cdot x^3 \cdot x^3$	$x^{12}$
$(7x^4)^3$	$7x^4 \cdot 7x^4 \cdot 7x^4$	$7^3 x^{12}$

### Summary:

To simplify powers of powers, multiply the exponents.

### Zero Power

Zero Power	Evaluate	Rule
$5^0$	1	$a^0 = 1$ ↑ anything
$7^0$	1	
$54^0$	1	

$$\frac{3^2}{3^2} = 3^{2-2} = 3^0$$

$$\frac{9}{9} = 1$$

↓

↖

## Negative Power

Quotient of Powers	Product Form	Fraction Form	Power Form
$2^2 \div 2^5$	<del><math>2 \times 2</math></del> <del><math>2 \times 2 \times 2 \times 2 \times 2</math></del>	$\frac{1}{2^3}$	$2^{-3}$
$3^3 \div 3^5$		$\frac{1}{3^2}$	$3^{-2}$
$5^3 \div 5^6$		$\frac{1}{5^3}$	$5^{-3}$
$a^2 \div a^3$		$\frac{1}{a}$	$a^{-1}$
$\frac{x^3y}{x^2y^2}$		$\frac{x}{y}$	$xy^{-1}$

Rule:

$$a^{-n} = \frac{1}{a^n} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

⊗ All final answers must have a positive exponent.

Exponent Rules

**Product** → multiplication

$$a^n \times a^m = a^{n+m}$$

**Quotient** → division

$$a^n \div a^m = a^{n-m}$$

**Power of a Power**

$$(a^n)^m = a^{n \times m}$$

**Zero Exponent**

$$a^0 = 1$$

**Negative Exponent**

$$a^{-n} = \frac{1}{a^n} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

**Power of a Product**

$$(ab)^n = a^n b^n$$

**Power of a Quotient**

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

**On the Boards...**

1. Simplify, but do not evaluate.

a)  $2^3 \times 2^4 = 2^7$       b)  $3^1 \times 3^{-4} = 3^{-3} = \frac{1}{3^3}$       c)  $(1.05)^{-3} \times (1.05)^4 = 1.05$

d)  $c^5 c^4 = c^9$       e)  $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^8$       f)  $a^4 a^{-2} a = a^3$

2. Simplify, but do not evaluate.

a)  $4^5 \div 4^2 = 4^3$       b)  $\frac{5^3}{5^7} = 5^{-4} = \frac{1}{5^4}$       c)  $(1.02)^{13} \div (1.02)^{10} = 1.02^3$

d)  $\frac{d^5}{d} = d^4$       e)  $(-3)^7 \div (-3)^{-4} = (-3)^{11}$       f)  $\frac{h^{30}}{h^{20}} = h^{10}$

3. Simplify, but do not evaluate.

$$\text{a) } (5^3)^2 = 5^6$$

$$\text{b) } (3^{-2})^5 = \frac{1}{3^{10}}$$

$$\text{c) } [(-2)^{-4}]^3 = \frac{1}{2^{12}}$$

$$\text{d) } (m^5)^4 = m^{20}$$

$$\text{e) } (r^{-10})^{-2} = r^{20}$$

$$\text{f) } (a^3)^3 = a^9$$

4. Evaluate without a calculator.

$$\text{a) } 10^4 = 10\,000$$

$$\text{b) } 9^0 = 1$$

$$\text{c) } 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\text{d) } 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\text{e) } \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{f) } \left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{1}\right)^2 = 25$$

6. Simplify each expression.

Which exponent laws did you use?

$$\text{a) } d^5 d^{-2}$$

$$= d^3$$

$$\text{b) } (x^{-5})^2$$

$$= \frac{1}{x^{10}}$$

$$\text{c) } \frac{c^{11}}{c^{-3}}$$

$$= c^{14}$$

$$\text{d) } \left(\frac{1}{z^3}\right)^{-6}$$

$$= z^{18}$$

$$\text{e) } n^4 n^{-13} n^7$$

$$= \frac{1}{n^2}$$

$$\text{f) } w^{-8} (w^3)^2$$

$$= \frac{1}{w^2}$$

$$\text{g) } \frac{s^5 s^4}{s^{-3}}$$

$$= s^{12}$$

$$\text{h) } \frac{(t^4)^{-5}}{t^6}$$

$$= \frac{1}{t^{26}}$$

8. Computer power has been doubling approximately every 2 years as more and smaller transistors have been integrated to build better computer chips. The number of transistors,  $T$ , in a chip has increased according to  $T = 4500(1.4)^n$ , where  $n$  is the number of years since 1974. Determine the number of transistors in a computer chip in each year.

a) 1974

b) 1972

c) 2002

$$n = 0$$

$$n = -2$$

$$n = 28$$

$$T = 4500(1.4)^0$$

$$T = 4500(1.4)^{-2}$$

$$T = 4500(1.4)^{28}$$

$$= 4500$$

$$= 2295.9$$

$$= 55564513.1$$

**Practice:***Handout*

1. Simplify the following to one power:

a)  $(x^4)(x^8)$

b)  $(b^{-3})(b^7)(b^2)$

c)  $\frac{c^4}{c^3}$

d)  $\frac{x^3}{x^{-2}}$

$$= x^{12}$$

$$= b^6$$

$$= c$$

$$= x^5$$

e)  $(x^5)^8$

f)  $(9b^5)^3$

g)  $(2y^{-3})^3$

h)  $(4z^3)(2z^7)$

$$= x^{40}$$

$$= 9^3 b^{15}$$

$$= 2^3 y^{-9}$$

$$= 8 z^{10}$$

$$= \frac{2^3}{y^9}$$

2. Simplify the following and write as a single power:

$$\text{a) } \frac{x^7 x^{-2}}{x^3}$$

$$= \frac{x^5}{x^3}$$

$$= x^2$$

$$\text{b) } \frac{b^{12}}{b^3 b^6}$$

$$= \frac{b^{12}}{b^9}$$

$$= b^3$$

$$\text{c) } \frac{y^7 y^{11}}{y^8 y^2}$$

$$= \frac{y^{16}}{y^6}$$

$$= y^{10}$$

$$\text{d) } \frac{-(-x)^2}{x}$$

$$= -\frac{x^2}{x}$$

$$= -x$$

$$\text{e) } (x^5)^4 \times (x^{-6})^{-2}$$

$$= x^{20} x^{12}$$

$$= x^{32}$$

$$\text{f) } \frac{(x^5)^2}{(x^2)^3}$$

$$= \frac{x^{10}}{x^6}$$

$$= x^4$$

$$\text{g) } \frac{(c^2)^3 (c^4)^2}{c^3}$$

$$\frac{c^6 c^8}{c^3}$$

$$= c^{11}$$

$$\text{h) } \frac{x^2 y^3 z}{x^3 y z}$$

$$= x^{-1} y^2 z$$

$$= \frac{y^2 z}{x}$$

3. Simplify and then evaluate for  $x=2$ ,  $y=-1$ ,  $z=3$

$$\text{a) } \frac{x^3 y^4 z^3}{x^2 y^2 z^2}$$

$$= x^3 y^2 z$$

$$\text{b) } \frac{x^6 y^{-3} z^2}{x^{-3} y z}$$

$$= \frac{x^9 z}{y^4}$$

$$= x^3 z$$

$$\text{c) } \frac{xy^4 z^4}{x^{-2} y^4 z^3}$$

$$\text{d) } \frac{(x^4)^3 (y^5)^2}{(x^4)^3 (y^2)^2}$$

$$= \frac{x^{12} y^{10}}{x^{12} y^4}$$

$$= y^6$$



*Seatwork*

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