Rational Exponents

Learning Goals

- understand the meaning of fractional exponents
- use fractional exponents to simplify and evaluate expressions

Use your TI-Nspire to EVALUTE

$1^2 =$ 1	$1^{\frac{1}{2}} =$ 1	$\sqrt{1} =$ 1
$2^2 =$ 4	$4^{\frac{1}{2}} =$ 2	$\sqrt{4} =$ 2
$3^2 =$ 9	$9^{\frac{1}{2}} =$ 3	$\sqrt{9} =$ 3
$4^2 =$ 16	$16^{\frac{1}{2}} =$ 4	$\sqrt{16} =$ 4

$1^3 = 1$ $1^{\frac{1}{3}} = 1$ $\sqrt[3]{1} = 1$

$2^3 = 8$ $8^{\frac{1}{3}} = 2$ $\sqrt[3]{8} = 2$

$3^3 = 27$ $27^{\frac{1}{3}} = 3$ $\sqrt[3]{27} = 3$

$4^3 = 64$ $64^{\frac{1}{3}} = 4$ $3 \rightarrow \sqrt[4]{64} = 4$

Rule:

$x^{\frac{1}{n}}$ MEANS $\sqrt[n]{x}$

What about ...

$\sqrt[3]{-27} = -3$ $\sqrt{-16} =$ Error?

$(+4)(+4) = +16$

$(-4)(-4) = +16$

Rule: odd $\sqrt{}$ negative $= -$

even $\sqrt{}$ negative = imaginary answer $\Rightarrow 4i$

Rational - Fraction

Rational Exponent - Exponent is a fraction

Radical - "root" $\Rightarrow \sqrt{}$

Ex. Change to a radical and evaluate

$16^{\frac{1}{2}} = \sqrt[2]{16}$

$= 4$

$1296^{\frac{1}{4}} = \sqrt[4]{1296}$

$= 6$

Ex. Change to a rational exponential form and evaluate

$\sqrt[4]{81} = 81^{\frac{1}{4}}$

$= 3$

$\sqrt[5]{32} = 32^{\frac{1}{5}}$

$= 2$

What happens if there is a numerator other than 1?

$8^{\frac{2}{3}} =$

Break up the exponent $8^{2 \cdot \frac{1}{3}}$

What rule does that look like? $(8^2)^{\frac{1}{3}}$

power of a power

$(8^2)^{\frac{1}{3}} = 64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

Rule:

$x^{\frac{m}{n}}$ MEANS $\sqrt[n]{x^m}$

Example: Simplify and then evaluate:

a) $64^{\frac{1}{2}}$

$= \sqrt{64}$

$= 8$

b) $8^{\frac{2}{3}}$

$\sqrt[3]{8^2}$

$\sqrt[3]{64}$ $\quad 2^2$

$= 4 \quad = 4$

c) $16^{\frac{3}{4}}$

$= \sqrt[4]{16^3}$

$= 2^3$

$= 8$

d) $4^{\frac{5}{2}}$

$\sqrt[2]{4^5}$

$= 2^5$

$= 32$

Solve for x.

$x^{\frac{1}{2}} = 5$

$\sqrt{x} = 5$

$x = 25$

$x^{\frac{1}{3}} = 4$

$\sqrt[3]{x} = 4$

$x = 4^3$

$x = 64$

$x^{\frac{2}{3}} = 16$

$\sqrt[3]{x^2} = 16$

$x^2 = 16^3$

$x = \sqrt[2]{16^3}$

$x = 4^3$

$x = 64$

On the Boards...

2. Evaluate each expression without using a calculator.

a) $9^{\frac{1}{2}}$

$= \sqrt{9}$

$= 3$

b) $49^{\frac{1}{2}}$

$= \sqrt{49}$

$= 7$

c) $64^{\frac{1}{2}}$

$= \sqrt{64}$

$= 8$

d) $27^{\frac{1}{3}}$

$= \sqrt[3]{27}$

$= 3$

e) $(-8)^{\frac{1}{3}}$

$= \sqrt[3]{-8}$

$= -2$

f) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$

5. Rewrite using radicals and evaluate without a calculator.

a) $32^{\frac{1}{5}}$ $= \sqrt[5]{32}$ $= 2$

b) $81^{\frac{1}{4}}$ $= \sqrt[4]{81}$ $= 3$

c) $16^{\frac{3}{2}}$ $= \sqrt{16^3}$ $= 4^3$ $= 64$

d) $9^{\frac{5}{2}}$ $= \sqrt{9^5}$ $= 3^5$ $= 243$

e) $100^{\frac{3}{2}}$ $= \sqrt{100^3}$ $= 10^3$ $= 1000$

f) $16^{\frac{3}{4}}$ $= \sqrt[4]{16^3}$ $= 2^3$ $= 8$

g) $8^{\frac{4}{3}}$ $= \sqrt[3]{8^4}$ $= 2^4$ $= 16$

h) $27^{\frac{3}{3}}$ $= \sqrt[3]{27^3}$ $= 27$

6. Use the table of values and graph of $y = 2^x$ shown here.

a) Explain why the value of $2^{\frac{1}{2}}$ must be between 1 and 2.

b) Use the graph to estimate the value of $2^{\frac{1}{2}}$ to the nearest tenth. 1.4

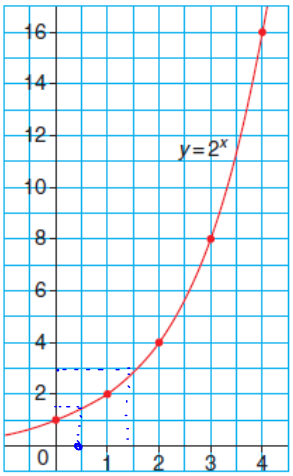
c) Which two whole numbers is $2^{\frac{3}{2}}$ between? Repeat for $2^{\frac{5}{2}}$ and $2^{\frac{7}{2}}$.

2 and 3 5 and 6

d) Estimate the value of $2^{\frac{3}{2}}$, $2^{\frac{5}{2}}$, and $2^{\frac{7}{2}}$ to the nearest tenth.

2.8 5.5

x	$y = 2^x$
0	1
1	2
2	4
3	8
4	16



7. The equation $P = 100(0.87)^x$ models the percent, P, of caffeine left in your body x hours after you consume it. Determine the value of P after each time.

a) $\frac{1}{2}$ h b) $\frac{3}{2}$ h c) 40 min

How do you know your answers are reasonable?

a.) $100(0.87)^{0.5}$

$= 93.27$

b.) $100(0.87)^{1.5}$

$= 81.15$

c.) $100(0.87)^{\frac{40}{60}}$

$= 100(0.87)^{\frac{2}{3}}$

$= 91.13$

Seatwork

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