Applications of Exponential Equations

Learning Goals
- solve exponential equations by
  1. Systematic Trial and Error
  2. Graphing

(6.7) Applications of Exponential Equations

Example: Population Growth

A biologist is researching a newly-discovered species of bacteria that grows at a constant rate of 0.25/hour. At time t=0 hours, he puts one hundred bacteria into a growth medium. Six hours later, he measures 450 bacteria. Since bacteria cultures grow exponentially, this problem can be modeled with the equation:

$A = Pe^{kt}$ where

A is the population size (in number of bacteria)
P is the original population size (in number of bacteria)
e is a constant = 2.7
k is the growth constant
t is the growth time (hours)

Thus, this situation can be modeled with the equation:

$$A = 100(2.7)^{0.25t}$$

How long with it take for this bacteria culture to reach 200 000 bacteria?

$$200\,000 = 100(2.7)^{0.25t}$$

$$\frac{200\,000}{100} = \frac{\cancel{100}(2.7)^{0.25t}}{\cancel{100}}$$

$$2000 = 2.7^{\underbrace{0.25t}}$$

Trial and Error

$$2000 = 2.7^{\boxed{7.65}}$$

$$0.25t = 7.65$$

$$t = 30.6$$

$\therefore$ it will take 30.6 h

Example 2: **Height of a Bouncing Ball**

Steve Gerrard kicks a soccer ball and measures the height of each bounce. His initial kick is 10 m high and the ball's height decreases at a rate of 60%. The height of the ball can be modeled with the equation:

$h = 10(0.6)^n$ where

h is the height of the ball in metres.
10 is the initial height of the ball
0.6 is the rate the height decreases with each bounce
n is the number of bounces

After how many bounces will the ball reach a maximum height of 4m?

$$4 = 10(0.6)^n$$

$$0.4 = 0.6^n$$

Trial and Error

$$0.4 = 0.6^{\boxed{1.79}}$$

$\therefore$ 2 bounces

1. Graph the equation on the grid provided.

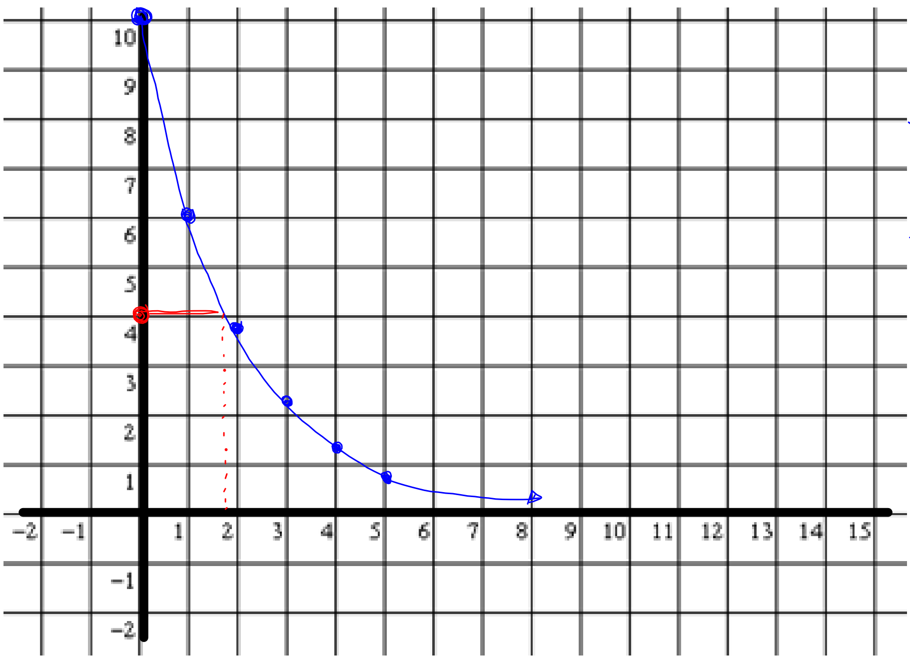


table of values

n	h
0	10
1	6
2	3.6
3	2.16
4	1.29
5	0.77

2. Draw a horizontal line across from h = 4 until it intersects the curve

3. Find the n-coordinate at this intersection point

Is your solution accurate?

yes

Example 3: Compound Interest

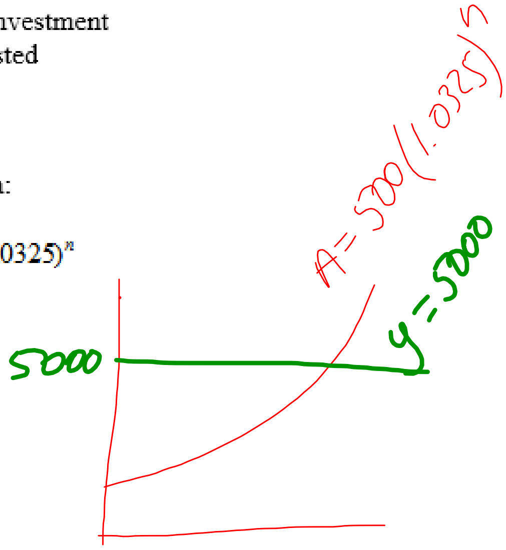
Marie is saving up to buy a car. She has $500 dollars to invest. She has found a high interest savings account with an interest rate of 3.25%. Compound interest is modeled using the formula:

$$A = P(1+i)^n \text{ where:}$$

A is the future amount of the investment
P is the original principle invested
i is the interest rate
n is the time invested

Thus, this situation can be modeled using the formula:

$$A = 500(1+0.0325)^n$$



1. How much money will Marie have saved after 3 years?

$$A = 500(1+0.0325)^3$$
$$= 500(1.0325)^3$$
$$= 500(1.1007)$$
$$= 550.35$$

2. How long will it take Marie to save $5000 to buy a car?

$$5000 = 500(1+0.0325)^n$$
$$10 = (1+0.0325)^n$$
$$10 = 1.0325^n$$

Trial and Error

$$10 = 1.0325^{\boxed{71.99}}$$

$\therefore$ 72 years

Practice questions are to be handed in on day 9.