Annuities / Future Value

Learning Goals
- **defining an annuity**
- **understand how annuities work**
- **use the annuity formula to solve real life problems**

Annuity - series of payments or deposits made at regular intervals

Simple Annuity - payment interval is the same as the compounding period

Ordinary Annuity - payment/deposit is made at the END of each interval

Matthew deposits $400 every month for 6 months. The money earns 6% interest per year, compounded monthly.

a. How much money will he deposit in total?

$400(6) = 2400$

b. How much money will he have at the end of 6 months?

months	1	2	3	4	5	6
$	400	400	400	400	400	400

↳ no interest 400

$400\left(1+\frac{0.06}{12}\right)^1$

$400\left(1+\frac{0.06}{12}\right)^2$

Total

Future Value of an Annuity (sum)

$$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$$

- R: deposit at intervals
- n: number of deposits
- i: interest / compounding period

$$FV = 400\left[\frac{\left(1+\frac{0.06}{12}\right)^{6}-1}{\frac{0.06}{12}}\right]$$

$$= 2430.20$$

∴ He has $2430.20

Thomas and Emily are saving up for retirement.

Thomas deposits $1000/year for 40 years.
Emily deposits $2000/year for 20 years.

Both earn 8% interest per year compounded annually.

Who has a better retirement plan?

Thomas

$$FV = 1000\left[\frac{\left(1+\frac{0.08}{1}\right)^{40}-1}{\frac{0.08}{1}}\right]$$

$$= 259056.52$$

Emily

$$FV = 2000\left[\frac{(1+0.08)^{20}-1}{0.08}\right]$$

$$= 91523.93$$

∴ Thomas has more $
∴ Investing for a longer time is better.

On the Boards...

1. Ashley deposits $500 every 6 months for 5 years. She gets an interest rate of 4% compounded semi-annually. How much money will she have at the end of 5 years?

$$FV = 500\left[\frac{\left(1+\frac{0.04}{2}\right)^{10}-1}{\frac{0.04}{2}}\right]$$

$$= 5474.86$$

1. Agnes starts an Education Savings Plan for her granddaughter. She deposits $200 every 3 months starting when the baby is 3 months old. Her interest is 5% compounded quarterly. How much money will accumulate by the time she is 18 years old?

$$FV = 200\left[\frac{\left(1+\frac{0.05}{4}\right)^{72}-1}{\frac{0.05}{4}}\right]$$

$$= 23134.72$$

1. Shannon deposits $3600 for 3 years at an interest rate of 6% compounded quarterly. Her friend Katie will deposit $100 at the end of each month for 3 years at 5% interest compounded monthly.
   1. Who will have more money?
   2. By how much?

Shannon

$A = P(1+i)^n$

$= 3600\left(1+\frac{0.06}{4}\right)^{12}$

$= 4304.23$

Katie

$FV = 100\left[\frac{\left(1+\frac{0.05}{12}\right)^{36}-1}{\frac{0.05}{12}}\right]$

$= 3875.33$

∴ Shannon has more money by $428.90

Seatwork

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