

Annuities / Present Value

- Learning Goals**
- defining a present value of an annuity
 - understand how annuities work
 - use the present value formula to solve real life problems

How is present value and future value different?

Future Value - you put the money in at regular intervals and you want to know how much you will have at the end.

ex. Educational savings plan

Present Value - you need to know how much money to put into the bank now if you want to take money out at regular intervals.

ex. OSAP gives you a lump sum in Sept. and you need to know how much you can take out each month

Present Value of an Annuity (sum)

$$PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

withdrawal at intervals (points to R)
 interest / compounding period (points to i)
 number of withdrawals (points to n)
 negative (points to the negative sign in the exponent)

Faith needs to have \$5000 for each year of college. Her bank pays 4% interest compounded yearly.

How much money does she have to put into the bank **today**, so she would have enough money for college?

↖ 3 year program

$$PV = 5000 \left[\frac{1 - \left(1 + \frac{0.04}{1}\right)^{-3}}{\frac{0.04}{1}} \right]$$

$$= 13875.46$$

∴ She needs \$13875.46

Abigail borrowed \$25000 to buy a car. Her interest rate is 2% monthly.

If she is paying it back monthly for 5 years, what will be her payment?

↑
R

$$25000 = R \left[\frac{1 - \left(1 + \frac{0.02}{12}\right)^{-5(12)}}{\frac{0.02}{12}} \right]$$

$$25000 = R (57.05)$$

$$438.19 = R$$

∴ monthly payment is \$438.19

Simple

$$I = Prt$$

Compounded

Future

$$A = P(1+i)^n$$

Present

$$PV = A(1+i)^{-n}$$

Annuity

Future

$$FV = R \left[\frac{(1+i)^n - 1}{i} \right]$$

Present

$$PV = R \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

Questions to ask before each problem.

1. Simple or compounded?
2. Present or Future?

borrow

pay back



3. Annuity or not?

On the Boards...

Tam is setting up an income fund for her retirement. She wishes to receive \$1500 every month for the next 20 years, starting 1 month from now.

The income fund pays 6.25% per year compounded monthly.

How much must Tam deposit now to pay for the annuity?

$$PV = 1500 \left[\frac{1 - \left(1 + \frac{0.0625}{12}\right)^{-240}}{\frac{0.0625}{12}} \right]$$

$$= 205218.51$$

$$\therefore \$205218.51$$

Nick borrowed money to buy a new car. His loan shark offered him a rate of 12% compounded semi-annually for 4 years. He has to pay back \$17190 or he will have his legs broken. What was the value of the **initial loan?**

NOT an ANNUITY!

$$PV = 17190 \left(1 + \frac{0.12}{2}\right)^{-8}$$

$$= 10785.22$$

\therefore initial loan was \$10785.22

A contest offers a prize of \$1000 every month for 1 year. The first payment will be made 1 month from now. If money can be invested at 8% per year compounded monthly, what cash payment received immediately is equivalent to the annuity?

$$PV = 1000 \left[\frac{1 - \left(1 + \frac{0.08}{12}\right)^{-12}}{\frac{0.08}{12}} \right]$$

$$= 11495.78$$

\therefore \$11495.78

Assessment Focus Isabel receives a disability settlement.

She must choose one of these payment plans.

- A single cash payment of \$80 000 to be received immediately
- Monthly disability payments of \$1200 for 10 years

Assume that money can be invested at 4.8% per year compounded monthly. Which settlement do you think Isabel should accept?

Justify your answer.

$$PV = 1200 \left[\frac{1 - \left(1 + \frac{0.048}{12}\right)^{-120}}{\frac{0.048}{12}} \right]$$

$$= 114\,187.00$$

∴ Take the monthly payments.

Seatwork

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