Exponential Models

Nov 6-3:40 PM

Bacteria Growth

A scientist has discovered a new strain of bacteria. The bacteria culture initially contained 1000 bacteria and the bacteria are doubling every hour.

a. Complete the chart for the first five hours:

Time intervals	begin	1 hr	2 hr	3 hr	4 hr	5 hr	6 hr	7 hr	8 hr	9 hr
Bacteria present	1000	2000	4000	8000	16000	32000	64000	128000	256000	512000
1st diff.	1000	2000	4000	8000	16000	32000				

1st diff.: 1000 2000 4000 8000 16000 32000 → not a straight line

2nd diff. 1000 2000 4000 8000 16000 → not quadratic

ratio $\frac{2000}{1000} = 2$ $\frac{4000}{2000} = 2$ $\frac{8000}{4000} = 2$ → exponential

Oct 19-7:49 AM

Exponential Models

$y=ab^x$

example: $y=4(2)^x$

- in a table of values the 1st and 2nd differences have a common ratio

Nov 6-3:40 PM

(5.5) Exponential Models

Linear and Exponential Models

Linear Models

- Recall a linear model represents quantities that change a constant rate.
- This occurs when a fixed amount is added to the quantity at regular intervals.

Exponential Models

- An exponential model represents quantities that change at a constant rate.
- This occurs when a fixed amount it multiplied by the quantity at regular intervals.

EXAMPLE:
Last year, a school had a population of 1000 students. This year, 1100 students attend the school, an increase of 100 students or 10%.

We can view this situation in two ways:

SCENARIO A: The population increases by 100 students each year.
SCENARIO B: The population increase by 10% each year.

Determine the population after 3 more years under each scenario:

	Student Population	
YEAR	Scenario A: Increase by 100	Scenario B: Increase by 10%
0	1000	1000
1	1100	1100
2	1200	1210
3	1300	1331

What type of growth is represented by each scenario?

•SCENARIO A:
–We repeatedly add 100 students.
–This is linear growth.

•SCENARIO B:
–We repeatedly multiply 1.10
–This is exponential growth.

Nov 25-10:54 AM

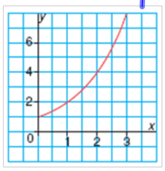
MAP 4C Unit 5, Lesson 3

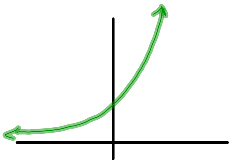
Identifying Exponential Models

•In a **table of values**, the growth/decay factors are constant

d	*P*	Growth factor
0	51.2	$64 \div 51.2 = 1.25$
1	64.0	$80 \div 64 = 1.25$
2	80.0	$100 \div 80 = 1.25$
3	100.0	

•The **graph** has an exponential **curve**.





•The **equation** is of the form

$y = ab^x$

Note: graph never crosses the x-axis.

y-int is always a $y=ab^x$

Where:

a is the y-int **value**
b is the growth decay **factor**

•For example,

$y = 10(2)^x$

Is an exponential function with an initial value of 10
And a growth factor of 2

Nov 25-10:54 AM

Comparing Pairs of Exponential Relations

Example:
Mr. Verberne scrapes 100 bacteria off from one seat in his class and 100 bacteria off another seat. He places each group in a separate petri-dish to grow.
Colony A doubles in size every hour.
Colony B triples in size every hour.

The growth of these bacteria can be modeled with the equations:

initial amount → y-int

COLONY A: $P = 100(2)^t$
COLONY B: $P = 100(3)^t$

growth rate

Graph these functions on the same screen using the graphing calculators.

Nov 25-10:54 AM

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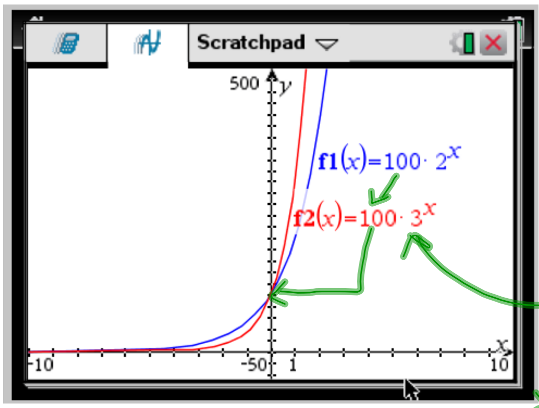


the larger the groth rate, the steeper the graph.

How are both graphs similar?
•They both have the same initial value (at 100 bacteria).
•This is because they both started with the same y-int.

How are both graphs different?
•The curve for Colony B increases much faster than for Colony A.
•This is because the growth factor for Colony B (which is 3) is larger than for Colony A (which is only 2).

Nov 6-4:07 PM

Exponential Growth

$$P_n=P_o(1+r)^n$$

Number of Growth Periods

Final Amount

Initial Amount

Rate of Growth (% must be changed to a decimal)

Example
- interest in the bank
- house prices

Mar 13-8:40 PM

Decay

$$P_{(n)} = P_o(1 - r)^n$$

time

final amount

initial amount

going down

rate of decay

Example
- car depreciates
- bacteria dies in response to antibiotics

Apr 18-10:59 AM

A car's initial value is $22000. It depreciates 10% each year. What will be the value 5 years later?

goes down

a. make a table of values

b. find an equation

c. use the equation to find the value 5 years later

years

x	y value
0	22000
1	19800
2	17820
3	16 038

×0.90 ×0.90 ×0.90

100% − 10%

$y = ab^x$

$y = 22000(0.9)^x$

$y = 22000(0.9)^5$

$= 12990.78$ ∴ 5 years later the value is \$12 990.78

Nov 6-4:15 PM

Examples:

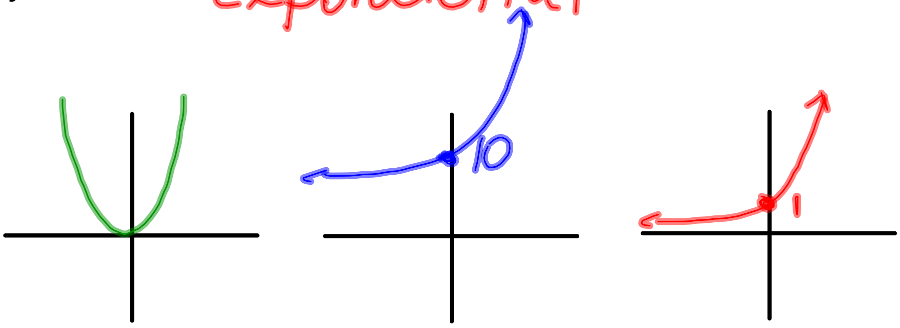
Which model represents an exponential function?

$y=10x^2$ quadratic

$y=10(2)^x$ exponential

$y=5^x$ exponential

1



Nov 6-4:08 PM

Seatwork

pg 315 # 1-8

Nov 6-4:18 PM