Optimizing Volume and Surface Area

Learning Goals

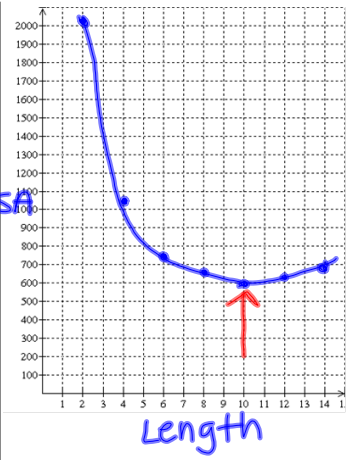
- Find optimal dimensions for 3D shapes

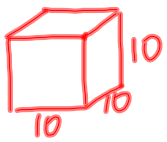
Aug 25-8:05 AM

You are asked to design a package to store gumballs and other assorted candies. The package must be a square-based prism, and it must hold a volume of 1000 cm^3. Since material costs money, you want to design a package that will have the least possible surface area.

Complete the table, showing possible dimensions for 7 square-based prisms with a **volume of 1000 cm^3**. ***Round to the nearest whole number.***

Base Length (cm)	Height (cm)	Surface Area (cm^2)
2	$\frac{1000}{2^2} = 250$	$SA = 2s^2 + 4sh$ $SA = 2(2)^2 + 4(2)(250)$ $SA = 2008\ cm^2$
4	$\frac{1000}{4^2} = 63$	$SA = 2s^2 + 4sh$ $SA = 2(4)^2 + 4(4)(63)$ $SA =$ 1040
6	$\frac{1000}{6^2} =$	$SA = 2s^2 + 4sh$ $SA = 2(\ )^2 + 4(\ )(\ )$ $SA =$ 744
8	$\frac{1000}{(8)^2} =$	$SA = 2s^2 + 4sh$ $SA = 2(\ )^2 + 4(\ )(\ )$ $SA =$ 640
10	$\frac{1000}{10^2}$	600
12	$\frac{1000}{12^2}$	624
14		672



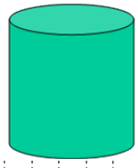


Graph the relationship between **base side length** and **surface area**. Use the base length as the independent variable. Draw a smooth curve and interpolate the base height of the prism with the **minimum surface area**. What are the dimensions of the square-based prism with the optimum surface area? CUBE

Nov 27-10:03 PM

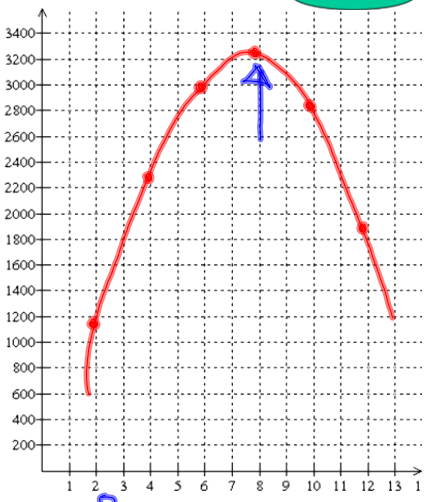
You are asked to design popcorn container. The package must be a cylinder, and it will be made of 1206 cm^2 of material, and you want it to hold the greatest possible volume.

SA



Complete the table, showing possible dimensions for 6 cylinders with a **surface area of 1206 cm^2**. ***Round to the nearest whole number.***

Radius (cm)	Height (cm)	Volume (cm^3)
2	$\frac{1206-2\pi(2)^2}{2\pi(2)} \doteq 94$	$V=\pi r^2 h$ $V \doteq \pi(2)^2(94)$ $V \doteq 1181$
4	$\frac{1206-2\pi(4)^2}{2\pi(4)} \doteq 44$	$V=\pi r^2 h$ $V \doteq \pi(4)^2(44)$ $V \doteq 2211$
6	$\frac{1206-2\pi(\ )^2}{2\pi(\ )} \doteq$	$V=\pi r^2 h$ $V \doteq \pi(\ )^2(\ )$ $V \doteq 2940$
8	16	3217
10	9	2827
12	4	1809



r = 8

h = 16

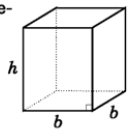
Graph the relationship between **radius** and **volume.** Use the radius as the independent variable. Draw a smooth curve and interpolate the radius of the cylinder with the maximum volume. What are the dimensions of the cylinder with the optimum volume?

2r = h d = h

Nov 27-10:03 PM

For a square-based prism with **given volume**, the **minimum surface area** will occur when …

Square-based prism



$SA = 4bh + 2b^2$

$V = b^2h$

cube

$V = b^2h$

$V = b^3$

Ex 1) A square-based prism must have a volume of 4410 cm^3. Determine the **dimensions** of the container that will minimize the surface area.

$V = b^3$

$\sqrt[3]{4410} = \sqrt[3]{b^3}$

$16.4 = b$

∴ dimension are 16.4cm × 16.4cm × 16.4cm

Jun 4-9:03 PM

For a square-based prism with **given surface area**, the **maximum volume** will occur when …

cube

$SA = 6b^2$

$\frac{SA}{6} = b^2$

$\sqrt{\frac{SA}{6}} = b$

Ex 2) A square-based prism must have a surface area of 1110 cm^2. Determine the **dimensions** of the container that will maximize the volume.

$b = \sqrt{\frac{SA}{6}}$

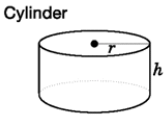
$= \sqrt{\frac{1110}{6}}$

$= 13.6$

∴ the dimensions are 13.6cm x 13.6cm x 13.6cm

Aug 25-8:16 AM

For a cylinder with **given volume**, the **minimum surface area** will occur when …



$SA = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

$d = h$

$2r = h$

$V = \pi r^2 h$

$= \pi r^2 (2r)$

$V = 2\pi r^3$

Ex 3) A cylinder must have a volume of 2650 cm^3. Determine the **dimensions** of the container that will minimize the surface area.

$V = 2\pi r^3$

$2650 = 2(3.14) r^3$

$\sqrt[3]{421.8} = \sqrt[3]{r^3}$

$7.5 = r$

∴ radius is 7.5 cm

height is 15cm

Nov 27-10:13 PM

For a cylinder with **given surface area**, the **maximum volume** will occur when …

$d = h$

$2r = h$

$SA = 2\pi r^2 + 2\pi r(h)$

$= 2\pi r^2 + 2\pi r(2r)$

$= 2\pi r^2 + 4\pi r^2$

$\boxed{SA = 6\pi r^2}$

Ex 4) A cylinder must have a surface area of 570 cm². Determine the **dimensions** of the container that will maximize the volume.

$SA = 6\pi r^2$

$570 = 6\pi r^2$

$30.2 = r^2$

$5.5 = r$

∴ radius is 5.5 cm

height is 11 cm

Aug 25-8:17 AM

Seatwork

Pg 110 # 2, 4, 11, 12, 15

Sep 9-9:19 AM